Tens and tenths: decimalising calendar and angle

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Humans tend to think in multiples of ten. Perhaps it's because we have ten fingers. Most major number systems throughout history, from the Roman to the Hindu-Arabic, are based around the number 10. Thus, it makes sense for the units we use in everyday life - whether they are used to represent currency, length, or time - to employ the number 10 as their base. We already count in tens; why not extend this to all measurements? This essay will explore the advantages and disadvantages of purely decimal systems, while also proposing a set of alternate, decimalised units for some quantities.

Non-decimal units of measurement are still used fairly widely, despite being phased out in most regions. The Imperial system, for example, is still used partially in the United Kingdom; a modified set known as US customary units, are kept alive in the United States of America. Under these systems, there are 12 inches in a foot, 3 feet in a yard, 22 yards in a chain, 10 chains in a furlong, 8 furlongs in a mile, and 3 miles in a league. This means that there are 5,280 feet in a mile. The difficulty in remembering so many different units, and their irregular ratios, puts anyone trying to use the Imperial system at quite a disadvantage.

The metric system, on the other hand, which is heavily decimalised, has one hundred centimetres per metre (equivalent to about 3.28 feet), one thousand metres per kilometre, one thousand kilometres per megametre, and so on. It uses the metre as a basic unit and simply adds prefixes that multiply the metre by a certain amount. That is, kilo stands for one thousand, while centi stands for one hundredth. This means that one doesn't have to memorise a plethora of different units.

This also makes it much easier to represent large distances in small units, or small distances in large units, with scientific notation. For example, 2,8750 kilometres can be written as 2.875×10^7 metres[1]. Large quantities are frequently represented this way in physics and the other sciences. If one were to do this in the Imperial system, however, one would have to make the conversion from miles to feet. 2,8750 miles would be 1.518×10^8 feet.[2]

The same advantages hold for decimal currency, which is why it was adopted in Britain and later on in the European Union, as the euro.

Decimalisation of temperature has already been undertaken; the Celsius (or Centigrade) system divides the temperature range at which water can maintain a liquid form (at sea level, that is) into 100 degrees, with water boiling at 100 degrees C and freezing at 0 degrees C. Kelvin retains the magnitude of the degree C, but instead moves 0 to absolute zero (so that the freezing point of water

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[1] 28750 km × 10^3 m = 28750000 m = 2.875×10^7 m

[2] 5280 feet × 28750 miles = 1.518×10^8 feet

is 273.15 K and its boiling point is 373.15 K). The SI units for mass (kilogram), current (ampere), frequency (hertz), energy (joule), and so on, are already decimalised (Chabay and Sherwood). Two quantities that have not been decimalised, however, are time and angular size.

The length of the year is, short of altering the Earth's orbit, fixed. A year contains 365 days, and is a practical natural unit as it is approximately the length of the Earth's orbital period around the Sun. It is easily measured; many of the activities humans study and rely on are by necessity set to a yearly clock, such as agriculture. However, 365 is not a multiple of 10. This renders a decimal calendar, or at least one which contains months of equal lengths, quite impractical. We could instead leave the current system of months in place, but complement it with a ten-day week. There would be 36.5 weeks in a year (36.6 on leap years).

If a ten-day week were introduced, a three-day weekend could be practical. The ratio of work days to weekend days would be 7:3, which is slightly less than the current ratio of 5:2. This could result in lowered productivity; an argument could also be made that more rest would equal more productivity. This essay will not involve itself in the complex social and economic factors involved in such a decision. Having been forced to accept that we cannot separate a year perfectly into multiples of 10, perhaps there would be little point in introducing a ten-day week.

The day, as another natural unit of time defined by the movement of the Earth, should be left as it is. However, it could be subdivided decimally. Each day and night can be divided into 10 "decimal hours", which in turn are divided into 100 "decimal minutes". Each minute is then divided into 100 "decimal seconds". There would hence be 100,000 seconds in the day[3]. There are 43,200 "real" (SI) seconds in the current (12-hour) day by comparison[4]; hence a decimal second would be 0.432 SI seconds long[5]. A decimal minute is hence 43.2 SI seconds long[6], or 0.72 current minutes[7]. A decimal hour is then 72 real minutes[8], or 1.2 real hours[9]. Whether this system would offer any real advantage is a matter for debate.

A clock-face would have the numbers 1 to 10 evenly spaced around its circumference. Each of these intervals would be separated into ten smaller intervals, representing the minutes. Like a modern clock-face, there would be a minute hand, an hour hand, and a second hand. This design has a slight advantage: as the hour hand moves between hours, the sub-intervals (representing minutes for the minute hand) it passes over correspond to ten minutes each. That is, as the minute hand moves through one major interval, the hour hand moves through one minor interval. So here, the minute markers represent both one minute (for the minute hand) and ten minutes (for the hour hand), and correspond directly to the numbered demarcations. On 12-hour clocks this does not occur, as there are 5 markers between each number rather than 12. Again, this symmetry provides only a slight advantage, but it might be helpful when teaching children to tell the time.

One disadvantage is that there is no major demarcation on the clock face for the quarter hour. A quarter of an hour would be 25 minutes, lying between the 2 and 3 marks. Quarter hours are often used as a convenient division of time; instead, fifth-hours might be used.

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[3] 100 seconds 100 minutes × 10 hours = 100,000 seconds
[4] 60 seconds * 60 minutes × 12 hours = 43,200 seconds
[5] 43200 / 100000 = 0.432
[6] 0.432 × 100 = 43.2
[7] 43 / 60 = 0.72
[8] 0.72 × 100 = 72
[9] 72 / 60 = 1.2

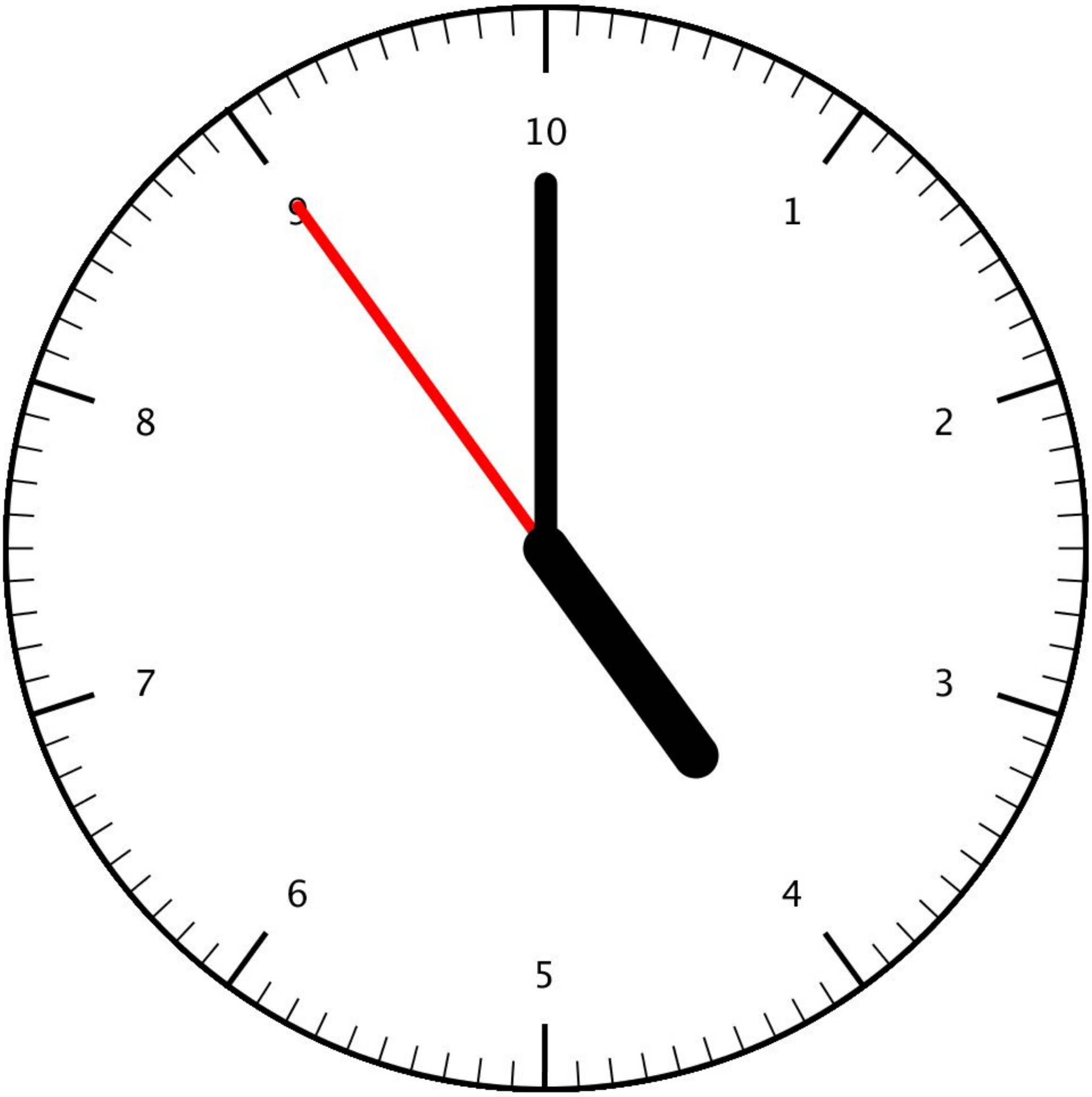

Figure 1: An example of a decimal clock-face, displaying 4 o'clock and 90 seconds, which corresponds to 4:48:54 on a 12-hour clock[10].

As for angular measure, the circle would be divided into 1000 decimal degrees to preserve the resolution of the degree. On one hand, the protractor would become quite crowded and more difficult to read, with almost three times as many markers. Standard protractors with a larger radius would become necessary, in order to space out the lines. On the other hand, angles could be measured with a greater resolution. Degrees would be subdivided into 10 or 100 “sub-degrees” for fine measurement, analogous to arcminutes and arcseconds in the current system.

However, under a thousand-degree system, significant angles such as 60 degrees ($\pi/3$ radians) and 30 degrees ($\pi/6$ radians) would not be integers. In a thousand-degree compass, $\pi/3$ radians would be 166.67 degrees[11] (to 2 decimal places), while $\pi/6$ radians would be 83.33 degrees[12]. Using these in calculations would be clunky and, unless the improper fractions 500/3 and 250/3 were to be

[10] 12/10 = 1.2, 1.2 × 4 = 4.8, 0.8 × 60 = 48, 60/10 = 6, 6 × 9 = 54
[11] 1000 / 6 = 166.67 to 2 decimal places
[12] 1000 / 12 = 83.33 to 2 decimal places

used, not perfectly accurate. 60 and 30 degrees are much easier to use and remember. These angles have a good deal of significance in trigonometry, and are used frequently in association with the sine and cosine functions (Hughes-Hallett). Other significant angles would not be affected, such as 180 degrees (π radians, 500 decimal degrees) and 90 degrees ($\pi/2$ radians, 250 decimal degrees).

Possibly, one could instead construct a system for angular measurement around the 30 degree base. $\pi/6$ radians could be made to equal ten degrees, each of which is divided into ten sub-degrees. $\pi/3$ radians would then be 20 degrees, $\pi/2$ would be 60 degrees, and a full revolution would be 120 degrees. The resolution is preserved by the sub-degree unit.

No matter how the degree system is decimalised, however, mathematicians will always prefer the radian as the unit of angular measure.

Rolling out decimal units across the board would require the redefinition of hundreds of concepts. The metre, a unit of length, is defined in terms of the second: specifically, it is defined as the distance travelled by light in a vacuum over 1/299,792,458 of a second (Chabay and Sherwood). If the second were redefined to the unit proposed in this essay, the metre would either be quite a different length, 0.432 of the current metre, or could be redefined to the distance travelled by light in a vacuum over 1/129,510,341 of a second, which would preserve its approximate length[13]. The speed of light, c, would then be 129,510,341 m/s. Other physical constants would also have to be redefined. Planck's constant, approximately 6.6×10^{-34} joule seconds (Chabay and Sherwood), ubiquitous in quantum physics, is defined in terms of the second.

Another example is the watt, a unit of power defined in terms of the second (specifically, it is the number of joules of energy changing form per second). Redefining the second would force us to redefine the watt, and to rewrite hundreds of physics and engineering textbooks.

So decimalisation of every unit of measurement starts to become somewhat impractical.

A brief tangent on alternatives to decimalisation:

Although decimal systems have become widely used, with most SI units being in some way decimalised, it is argued by some that a duodecimal system, that is, one which uses the number 12 as its base, would be preferable (Aitken). The number 12 is divisible by 2, 3, 4, and 6, as compared to the number 10 which is divisible only by 2 and 5. 144, which is 12 squared and would replace 100 in a duodecimal system, is divisible by 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48 and 72. It could also be significantly more efficient, and would make it easier for children to learn division and multiplication in (Aitken). Perhaps, then, a more fitting essay would be on how a duodecimal system could be implemented.

However, as mentioned before, the world is already heavily entrenched in its current numerical systems, and a base-12 system lacks the advantage of corresponding with the number of fingers on our hands. Completely overhauling not only our units, but the way we count, would cause confusion and meet resistance all over the world.

Such a transition is not without precedent, however. The change in Europe from Roman numerals to the (clearly superior) Hindu-Arabic system took place simply because it is easier to conduct calculations with the Hindu-Arabic "place" system (Aitken). Perhaps, then, if there were a great enough advantage in adopting a duodecimal or "dozenal" system, the world might embrace it. In this case, there would be little need to alter most units of time, as hours, minutes, and seconds of the day are already based around multiples of 12, as is the degree system of angular measurement.

[13] (1/299,792,458)/0.432 = 1/129510341.856

Some might argue that the decimalisation of all units would remove some of life's flavour. It would also increase efficiency and make calculations in many areas of science just a little bit easier. However, the decimalisation of degrees and certain units of time is not necessarily practical, because of the confusion it would cause. In the case of angular measure, a decimal system is actually less efficient. Current systems seem to be working fairly well, so if it ain't broke, why fix it?

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